

Angular momentum near the black hole threshold in scalar field collapse

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For the formation of a black hole in the gravitational collapse of a massless scalar field, we calculate a critical exponent that governs the black hole angular momentum for slightly non-spherical initial data near the black hole threshold. We calculate the scaling law by second-order perturbation theory. We then use the numerical results of a previous first-order perturbative analysis to obtain the numerical value $\mu \approx 0.76$ for the angular momentum critical exponent. A quasi-periodic fine structure is superimposed on the overall power law. [S0556-2821(99)00310-0]

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I. INTRODUCTION

In the space of initial data for a self-gravitating system, the black hole threshold is the limit between initial data that eventually form a black hole and initial data that do not. More precisely, if one considers smooth one-parameter families of initial data such that for large values of the parameter p a black hole is formed, but not for small values, then there is a critical value p_* (which can be found by bisection) where a black hole is first formed. Such data are called critical data. All numerical experiments are compatible with the picture that the critical data form a smooth hypersurface of codimension 1, called the critical surface, in the infinite-dimensional space of initial data.

The evolution of any data sufficiently close to the black hole threshold shows what is now commonly known as critical phenomena in gravitational collapse. The solution evolved from near-critical data, with $p \approx p_*$, approaches a universal intermediate attractor in a region of spacetime. This universal “critical solution” has a discrete or continuous self-similarity. Numerical experiments as well as perturbative calculations are compatible with the picture that it is an attractor inside the critical surface, with precisely one unstable perturbation mode pointing out of the critical surface.

If $p > p_*$, and a black hole is formed, its mass scales with p as [1]

$$M \approx C (p - p_*)^\gamma, \quad (1)$$

where γ is the same for all families of initial data. However, scaling is not only a supercritical property. Any dimensionful variable of the system scales with $p - p_*$. For example the maximum curvature of spacetime scales as $(p_* - p)^{-2\gamma}$ for $p < p_*$ [2]. The critical exponent γ is directly related to the growth rate of the one unstable mode.

The critical exponent γ is independent of the initial data. It depends on the type of matter (or absence of matter, in the

collapse of pure gravitational waves), but is independent of any dimensionful constants in the matter equations of motion. Critical phenomena were first observed in the spherically symmetric collapse of a scalar field [1] and the axisymmetric collapse of gravitational waves [3]. The structure of the intermediate attractor that plays the role of the critical point was clarified in [4] and [5]. The critical exponent for the black hole mass was calculated by perturbation theory around the critical point in [6] and more generally in [7]. An independent critical exponent for the black hole electric charge was predicted in [8] and subsequently measured in [9]. The prediction of a critical exponent for the black hole’s angular momentum required perturbation theory beyond spherical symmetry, and was first carried out in [10,11] for perfect fluid collapse. A detailed review of all this can be found in [12].

Now two of us have calculated the non-spherical linear perturbations around the critical point in scalar field collapse [13]. In the present paper, we build on that work to quantitatively predict the scaling of the black hole angular momentum in critical scalar field collapse. The perfect fluid and scalar field cases have in common that the critical spacetime is spherically symmetric. The black hole angular momentum, which breaks spherical symmetry, can therefore be treated perturbatively. There are two important differences, however.

The first difference is this. The critical point spacetime in perfect fluid collapse has, apart from spherical symmetry, another continuous symmetry, namely continuous self-similarity (CSS), also called a homothety. Roughly speaking, this means that it does not depend on t and r separately, but only through the dimensionless combination $x = -r/t$ (for a suitable choice of coordinates that we do not need to discuss here). The perturbations of this solution must then have the form $\text{Re}[(-t)^{-\lambda} f(x)]$, where both the eigenvalue λ and the mode function $f(x)$ can be complex. In the perfect fluid case the λ of the perturbation that dominates angular momentum—the $l=1$ axial mode with the largest real

part—is real. (The statement in [11] that λ is complex was wrong.) The black hole angular momentum in critical perfect fluid collapse scales as

$$\tilde{L} \simeq \tilde{L}_0 (p - p_*)^\mu \quad (\text{perfect fluid, CSS}). \quad (2)$$

In contrast, the scalar field critical spacetime depends not only on x , but in a restricted way also on t : it is periodic in $\tau = -\ln(-t)$. (Again this holds only in suitable coordinates.) This symmetry is called discrete self-similarity (DSS) [7]. Furthermore, the critical exponent itself is complex. The imaginary part of the critical exponent and the independent period Δ in τ of the background critical point solution then combine to give an overall power law modified by quasi-periodic behavior, as we shall see below.

The second new difference between the fluid and the scalar field is that angular momentum can be treated in first-order perturbation theory around spherical symmetry for perfect fluid collapse, but not for scalar field collapse. We have to go to second-order perturbation theory to obtain a non-zero effect. Fortunately the relevant second-order perturbation degree of freedom does not obey a wave-like equation which would require independent free initial data, but is totally determined by its source, which is quadratic in first-order perturbations. This allows us to obtain the desired result with a calculation based on dimensional analysis and selection rules for the angular dependence of the perturbations. At the end of the calculation we insert the numerical value for a particular complex eigenvalue λ into our result to obtain a numerical value for the angular momentum critical exponent.

The paper is structured as follows. First we define perturbation theory around spherical symmetry to all orders and show, both formally and intuitively, why we have to go beyond first-order perturbations. Then we consider the second-order perturbations that give rise to the angular momentum. In order to make our presentation more self-contained, we briefly review how the mass scaling law is derived. Then we derive the angular momentum scaling law for a spherical, DSS, background, and insert numbers for the scalar field case.

II. FIRST-ORDER PERTURBATIONS AROUND SPHERICAL SYMMETRY

In order to define higher order perturbation theory, we formally expand the metric and the scalar field as

$$g_{\mu\nu}(\epsilon) = g_{\mu\nu}^{(0)} + \epsilon g_{\mu\nu}^{(1)} + \epsilon^2 g_{\mu\nu}^{(2)} + O(\epsilon^3), \quad (3)$$

$$\phi(\epsilon) = \phi^{(0)} + \epsilon \phi^{(1)} + \epsilon^2 \phi^{(2)} + O(\epsilon^3). \quad (4)$$

In the following we use the shorthand u to denote both $g_{\mu\nu}$ and ϕ . If we write the field equations, that is, the Einstein equations and the scalar wave equation, formally as

$$\mathcal{E}(u) = 0, \quad u = (g_{\mu\nu}, \phi) = u(\epsilon), \quad (5)$$

and take formal derivatives with respect to ϵ , we obtain, as the leading orders,

$$\mathcal{E}(u^{(0)}) = 0, \quad (6)$$

$$\mathcal{L}(u^{(1)}) = 0, \quad (7)$$

$$\mathcal{L}(u^{(2)}) = S^{(2)}(u^{(1)}, u^{(1)}), \quad (8)$$

where \mathcal{L} is a linear derivative operator, and S is a quadratic derivative operator.

We perturb around a spherically symmetric solution $u^{(0)}$. In this paper we consider a double perturbation expansion, both around spherical symmetry and around criticality. The small parameter ϵ in the following always refers to deviations from spherical symmetry. (The small deviation from criticality will be parametrized by $p - p_*$.)

We begin by establishing that no black hole angular momentum can arise in first-order perturbation theory, for a spherically symmetric background solution with only scalar field matter. We begin by noting [14] that the Kerr metric with mass M and angular momentum L in Boyer-Lindquist coordinates can be written as a perturbation of the Schwarzschild metric (for simplicity we assume that the angular momentum is in the z -direction):

$$g_{\mu\nu}^{(0)} dx^\mu dx^\nu = - \left(1 - \frac{2M}{r} \right) dt^2 + \left(1 - \frac{2M}{r} \right)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2), \quad (9)$$

$$g_{\mu\nu}^{(1)} dx^\mu dx^\nu = -4 \frac{L}{r} \sin^2 \theta dt d\varphi = 4 \frac{L}{r} \sin \theta \frac{\partial}{\partial \theta} \sqrt{\frac{4\pi}{3}} Y_{10}(\theta, \varphi) dt d\varphi, \quad (10)$$

where Y_{10} is the spherical harmonic with $l=1$ and $m=0$. Note that this is the only first-order metric perturbation when we expand in powers of L . The other metric coefficients are changed only from $O(L^2)$ on.

We adapt the gauge-invariant perturbation framework of Gerlach and Sengupta [15], which is reviewed in more detail in [13]. In this framework, all four-dimensional tensor perturbations are covariantly decomposed into series of tensorial spherical harmonics, which carry all the angular dependence, with coefficients which depend on r and t . These harmonics are tensors derived from the Y_{lm} and their derivatives, residing on the two-spheres of spherical symmetry (coordinates θ and φ , tensor indices a). The coefficients of those expansions are tensors residing in the two-dimensional reduced manifold (coordinates r and t , tensor indices A). This decomposition is performed because around spherical symmetry different l, m components of the expansion decouple so we can study each case separately. Each l, m linear perturbation decouples further into two parts: axial [with parity $(-1)^{l+1}$] and polar [with parity $(-1)^l$].

In this notation, the linear metric perturbation (9) is written as

$$g_{\mu\nu}^{(1)} dx^\mu dx^\nu = 2k_A^{(1)} dx^A S_{10a} dx^a, \quad (11)$$

where S_{10a} is the axial harmonic vector field with angular dependence $l=1$, $m=0$, formed by the general rule $S_{lma} = \epsilon_a^b Y_{lm;b}$. (Here $:a$ is the covariant derivative on the unit two-sphere, and ϵ_{ab} is the corresponding unit antisymmetric tensor.) The angular momentum perturbation L_z we are interested in can therefore be characterized as the axial $l=1$, $m=0$, gauge-invariant metric perturbation. ($m=1$ and $m=-1$ parametrize L_x and L_y .) By comparison with Eq. (9) we read off that

$$k_A^{(1)} dx^A = \sqrt{\frac{4\pi}{3}} \frac{2L}{r} dt \quad (12)$$

for $l=1$ and $m=0$. Here $k_A^{(1)}$ is only partially gauge-invariant for $l=1$, but its curl $\Pi^{(1)}$ is gauge-invariant for all $l>0$, and contains all the gauge-invariant information. We calculate

$$\Pi^{(1)} = \epsilon^{AB} (r^{-2} k_A^{(1)})_{|B} = -\sqrt{\frac{4\pi}{3}} \frac{6L}{r^4}, \quad (13)$$

for $l=1$ and $m=0$. (In the notation of [13] these objects are simply called k_A and Π because there we only work with first-order perturbations.) A is the covariant derivative on the reduced manifold, and ϵ^{AB} the corresponding unit antisymmetric tensor.

Let us compare this particular solution with the general linearized equation (7) it must obey. For $l \geq 2$, $\Pi^{(1)}$ obeys a wave equation, but for $l=1$ it obeys a first-order differential equation that can be integrated trivially to yield

$$r^4 \Pi^{(1)} - 16\pi T^{(1)} - c = 0, \quad (14)$$

where $T^{(1)}$ is a scalar constructed from the axial gauge invariant matter perturbations, and c is an integration constant. Note that \mathcal{L} degenerates from a derivative operator to an algebraic operator for $l=1$ axial perturbations. In spite of its trivial appearance, Eq. (14) is the linearized Einstein equation relating gauge-invariant axial $l=1$ metric perturbations to their matter sources.

If the spacetime is vacuum, the spherical background must be Schwarzschild, and the matter perturbation $T^{(1)}$ vanishes. The only axial physical perturbation is then the one that takes the Schwarzschild metric infinitesimally into the Kerr metric. We see that, for this perturbation,

$$T^{(1)} = 0, \quad c = -6\sqrt{\frac{4\pi}{3}} L. \quad (15)$$

(There is one other physical perturbation of the Schwarzschild metric, $l=0$ polar, which changes the mass of the black hole.)

If we demand regularity at the center $r=0$, as we do here, we must have $c=0$, and this is also true in the presence of matter. The scalar field and its perturbation (to all orders) are polar, simply because they are spacetime scalars. Therefore $T^{(1)}$ vanishes for scalar field matter and we have $\Pi^{(1)}=0$. (To the next order, polar and axial perturbations do mix, so

that to second order T and hence Π do not vanish. This will be discussed in the next section.)

While this is a complete argument for the absence of black hole angular momentum in the first-order perturbation calculation, we have also found two intuitive ones. Angular momentum is present in the first-order perturbations of spherical fluid collapse, but not of scalar field collapse, because the fluid is made up of individual particles with a rest mass which can go round while the entire configuration remains axisymmetric. The angular momentum is proportional to the tangential velocity component u^φ . The scalar field has no such particles, and we need to make spoke-like structures in the field (thus breaking axisymmetry) and then make these go round. Intuitively, we need two powers of ϵ to do this: one to make spokes and another to make them go round. This argument is backed up by the observation that the conserved angular momentum (Noether charge) of a scalar field on a Minkowski background, for example the z -component, is simply

$$L_{\phi, \text{Minkowski}}^z = \int \phi_{,t} \phi_{,\varphi} d^3x = O(\epsilon^2). \quad (16)$$

One can easily see that this is again quadratic in deviations from spherical symmetry.

A second intuitive argument for the absence of first-order angular momentum perturbations in the scalar field comes precisely from its partial equivalence with a perfect fluid (with $p=\rho$): the 4-velocity of that pseudo-fluid is the normalized gradient of the scalar field and, therefore, irrotational.

III. SECOND-ORDER PERTURBATIONS

In second-order perturbation theory, we can define second-order versions of the gauge-invariant perturbations. (Equivalently, we could fix the gauge separately at each order.) Independent of the detailed field equations, the term in the n th order equations that is linear in the n th order perturbations is the same at all orders, namely \mathcal{L} . For $l=1$ axial perturbations, Eq. (8) therefore takes the simple algebraic form

$$r^4 \Pi^{(2)} - 16\pi T^{(2)} - c = S^{(2)}(u^{(1)}, u^{(1)}) \quad (17)$$

where $\Pi^{(2)}$ and $T^{(2)}$ are just $\Pi^{(1)}$ and $T^{(1)}$ with $u^{(1)}$ replacing $u^{(2)}$. Just as for the first-order perturbations, $T^{(2)}$ vanishes identically for scalar field matter, and the integration constant c vanishes if we consider only solutions with a regular center $r=0$.

But now there is also the source $S^{(2)}$, which is quadratic in first order perturbations $u^{(1)}$. It does not vanish in general, and generates a non-vanishing $\Pi^{(2)}$. Therefore it can introduce angular momentum. If and when a black hole is formed, it must settle down to the Kerr solution at late times. $S^{(2)}$ must then approach a constant at late times outside the horizon, thus transforming a Schwarzschild into a Kerr black hole. Inside the horizon, where the singularity forms, perturbation theory around a regular center will break down, but that does not affect our calculations.

Finding the expression for $S^{(2)}$ would require more effort than writing down the left-hand side, but fortunately we are interested only in scaling arguments, not in the detailed behavior of $\Pi^{(2)}$ as a function of r and t . We note that $\Pi^{(2)}$ belongs to the axial sector, and that we are only interested in its $l=1, m=0$ component, as that is the one connected to the black hole angular momentum. All other second-order perturbations (except a mass perturbation, as mentioned above) must eventually be radiated away as the black hole settles down to the Kerr solution. We can therefore restrict our attention to terms which are quadratic in first-order perturbations, axial, and have angular dependence $l=1, m=0$. The restriction to $m=0$ is equivalent to the restriction to the z -component, L_z , of angular momentum. We make it only for simplicity of presentation. L_x and L_y are related to complex linear combinations of $m=1$ and $m=-1$.

Let us denote the two factors $u^{(1)}$ by u' and u'' . They must have $m=m'+m''=0$. We do not require that the number $l'+l''$ be odd that of u' and u'' one be polar and the other axial. An example for the mixing of polar and axial perturbations to quadratic order is

$$Y_{lm} Y_{l-m;a} = -im \sqrt{\frac{3}{16\pi}} S_{10a} + \text{other terms} \quad (18)$$

for any l and m . Note that any $m'+m''=0$ gives rise to the axial vector field S_{10a} that characterizes angular momentum in the z -direction, except $m'=m''=0$. That $m'=m''=0$ is excluded is also clear on physical grounds, as we have argued above that an axisymmetric scalar field configuration cannot have angular momentum. In the simplest case $l=1$ the complete expression is

$$Y_{11} Y_{1-1;a} = -i \sqrt{\frac{3}{16\pi}} S_{10a} + \frac{1}{4\sqrt{5}\pi} Y_{20;a}. \quad (19)$$

In other words two polar $l=1$ perturbations combine to give an axial $l=1$ perturbation, as well as the expected $l=2$ polar perturbation.

IV. REVIEW OF MASS SCALING IN CRITICAL COLLAPSE

We have clarified the role of second-order perturbation theory for calculating angular momentum-like quantities in almost spherical scalar field collapse. In the following we specialize to a particular background solution, the so-called critical solution. The derivation of the angular momentum scaling law is a continuation of the previous derivations of scaling laws for the mass [4,7], electric charge [8] and angular momentum [11]. Therefore we do not try here to be fully self-contained, but rather remind the reader of the general ideas underlying these calculations. A more detailed review is contained in [12].

Critical collapse is dominated by a single solution which has the two crucial properties of being self-similar (CSS or DSS) and of having precisely one growing perturbation mode. This solution is best given in coordinates x and τ , which are (with some simplification) $x = -r/t$ and τ

$= -\ln(-t)$. The important properties of these coordinates are that both are dimensionless, that x is invariant under rescaling of space and time, and that τ , being the logarithm of a scale, changes only by an additive constant. The critical scalar field solution, in coordinates τ, x, θ, φ , is then of the form

$$g_{\mu\nu} = l_0^2 e^{-2\tau} \bar{g}_{\mu\nu}(x, \tilde{\tau}), \quad \phi = \phi(x, \tilde{\tau}), \quad (20)$$

where the tilde over the argument τ indicates that the conformal metric $\bar{g}_{\mu\nu}$ and scalar field ϕ depend on τ only periodically, with a period Δ . Note that $\bar{g}_{\mu\nu}$, ϕ , and the coordinates τ, x, θ, φ are conveniently thought of as dimensionless, with the arbitrary constant scale l_0 the only dimensionful quantity. Note also that by construction, l_0 always comes together with $e^{-\tau}$. From this it follows that, in the self-similar spacetime,

$$\text{any masslike quantity} \sim l_0 e^{-\tau}. \quad (21)$$

Now consider the one growing linear perturbation mode. As the background is periodic in τ , it must be of the form

$$\text{growing mode} \sim (p - p_*) e^{\lambda_0 \tau} f_0(x, \tilde{\tau}). \quad (22)$$

λ_0 is real and positive, so that the perturbation grows as we approach smaller scales ($t \rightarrow 0$, $r \rightarrow 0$, $\tau \rightarrow \infty$). As by assumption there is only one such mode, and the background is real, λ_0 and f_0 must be real, but in general λ_i and f_i exist in complex conjugate pairs. The overall amplitude of the growing mode depends on the initial data in a complicated way, but to leading order it must be proportional to $p - p_*$, because for $p = p_*$ the critical solution exists forever, by definition of being the critical solution, and so the growing mode cannot be present.

Now consider the spacelike hypersurface $\tau = \tau_*$, with the value of τ_* defined by

$$\text{growing mode} \sim (p - p_*) e^{\lambda_0 \tau_*} \sim \text{some fiducial amplitude}. \quad (23)$$

For τ_* this gives

$$\lambda_0 \tau_* = -\ln(p - p_*) + \text{const.} \quad (24)$$

At a later stage, we can no longer approximate the spacetime as self-similar plus a perturbation, but the Cauchy data at $\tau = \tau_*$ are independent of the initial data—the decaying perturbations have all decayed, and the one growing perturbation has reached its fiducial amplitude—up to an overall scale, which must be given by Eq. (21). Therefore we have, for the black hole mass,

$$M \sim l_0 e^{-\tau_*} \sim (p - p_*)^{1/\lambda_0}, \quad (25)$$

so that we have found the law (1) with $\gamma = 1/\lambda_0$.

V. ANGULAR MOMENTUM SCALING IN DSS CRITICAL COLLAPSE

At the fiducial time $\tau = \tau_*$, we have, for the angular momentum of what will become the black hole,

$$L_z \sim S^{(2)}(u^{(1)}, u^{(1)})(\tau_*) \sim l_0^2 e^{-2\tau_*} \text{Re } u'(\tau_*) \text{Re } u''(\tau_*) \\ \sim l_0^2 e^{-2\tau_*} \text{Re}[C' f'(\tau_*) e^{\lambda' \tau_*}] \text{Re}[C'' f''(\tau_*) e^{\lambda'' \tau_*}]. \quad (26)$$

Here λ' and λ'' are the complex exponents of the most slowly decaying modes of two separate first-order perturbations which are compatible with the angular dependence selection rules, and f' and f'' are the corresponding complex mode functions, which are periodic in τ with period Δ . C' and C'' are complex constants that depend on the family of initial data. (They also depend on p , but here we only take their leading order, which is constant. We take the real parts of u' and u'' separately because the complex notation is only a shorthand for sines and cosines.) In order to simplify we have made the jump from the x -dependent quantity $S^{(2)}$ to the simple number L in the first equality, and we have therefore suppressed the x -dependence of f' and f'' . Note that the factor $l_0^2 e^{-2\tau_*}$ in the second equality appears by dimensional analysis— L has dimensions (length)².

In [13] we found that the most slowly decaying perturbation with $l \neq 0$ is the polar $l=2$ perturbation, which has $\lambda \simeq -0.06(1/\Delta) + 0.30(2\pi i/\Delta) \simeq -0.017 + 0.55i$. Therefore L_z will be dominated by contributions where both u' and u'' are $l=2$ polar. This does not mean, however, that $u' = u''$. $S^{(2)}$ will contain dominant contributions from both $m' = -m'' = 1$ and $m' = -m'' = 2$. Furthermore, C' and C'' are independent complex constants for each of these two cases. (We have chosen our complex notation so that C can take arbitrary complex values, but λ and its complex conjugate are not counted as independent.) The function $L_z(p-p_*)$ is therefore parametrized by four independent complex constants, namely the constants C for $l=2$, polar, with $m = -1, 1, -2, 2$.

Inserting the value (24) for τ_* , and putting back the vector character of \vec{L} , and the periodic nature of the background, we can summarize our result as

$$\vec{L} \simeq \vec{L}_0[\ln(p-p_*)](p-p_*)^\mu \quad (\text{DSS}) \quad (27)$$

where $\vec{L}_0[\ln(p-p_*)]$ is a quasiperiodic function that depends on the family of initial data, and μ is a universal critical exponent that is given by

$$\mu = \text{Re} \frac{2 - \lambda' - \lambda''}{\lambda_0} = (2 - 2 \text{Re } \lambda') \gamma. \quad (28)$$

Note that λ' has negative real part, so that we have $\mu > 2\gamma$. With $\gamma \simeq 0.374$ and $\text{Re } \lambda' \simeq -0.017$ we predict a critical exponent $\mu \simeq 0.76$, which is barely larger than 2γ .

The Fourier spectrum of $\vec{L}_0[\ln(p-p_*)]$ with respect to its formal argument contains the angular frequencies

$$N \frac{2\pi\gamma}{\Delta}, \quad N \frac{2\pi\gamma}{\Delta} \pm 2\gamma \text{Im } \lambda' \quad (29)$$

for integer N . We could be more precise by writing down the general form of $\vec{L}_0[\ln(p-p_*)]$ as a sum involving eight universal (but, from the present calculation, unknown) periodic functions and 24 real constants depending on the family of initial data, but that would not be very helpful. Nevertheless, in numerical collapse simulations it should be possible to spot not only the overall power law, but also a fine structure with the fundamental angular frequency $2\pi\gamma/\Delta \simeq 0.683$ and the offset $2\gamma \text{Im } \lambda' \simeq 0.41$ in the Fourier transform of \vec{L}_0 with respect to $\ln(p-p_*)$.

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